figuration, it is clear that

$$1)\ I/AN_0\delta_\alpha - 2 \equiv a\delta_\alpha \qquad 2)\ I/AN_0\delta_\alpha \equiv b$$

$$3)\ 2/\delta_\alpha^2 \equiv c \qquad 4)\ (t - \delta_f)/AN_0^2\delta_\alpha^2 \equiv D$$

have to be individually and severally the same in the original and the model configurations. For example, the original engine may have a stable nodal point characteristic whereas the scaled engine may display small self-sustained oscillations if the changes in the inertias and the delays are not matched as required while an independent criterion determines the choice of rotational speed.

In general the model may have to be run at a different speed. The matching with respect to each of the parameters is essential in order to maintain similarity under dynamic conditions. The size, weight, rotational speed, and the delay times should, therefore, be so chosen in the model that such matching is assured and it is on this basis that the size of a test facility has to be considered for a given size of engine.

A more extensive analysis taking account of transients in individual components of the engine, and which is applicable to mixed cycles will be communicated elsewhere.

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Two-Dimensional Air-Cushion Vehicle Critical Forward Speeds

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Nomenclature

- h = vehicle clearance (see Fig. 2)
- H = jet total pressure (gage)
- p_c = cushion pressure (gage)
- M_T = total momentum flux from front and rear jets, per unit nozzle length for peripheral jet
- t = jet thickness at nozzle exit (see Fig. 2)
- t_c = jet thickness at station c (see Fig. 2)
- V = mean jet velocity at nozzle exit [defined in Eq. (1)]
- V_c = mean jet velocity at station c [defined in Eq. (2)]
- V_f = critical forward speed for peripheral jet
- θ = jet angle at nozzle exit (see Fig. 2)
- ρ = air density
- ψ = ratio of peripheral jet to plenum chamber critical forward speeds

Introduction

In recent years, a number of high-speed, guided land transport vehicles employing air-cushion support have been proposed, i.e., the Foa "Project Tubeflight,"[1] the Hovercraft Development Limited "Tracked Hovercraft,"[2] and the Bertin "Aerotrain."[3] The advantage of such support systems compared with conventional wheels is that the reduced "foot" pressures allow lower track construction and maintenance costs. Additionally, these systems remove frictional resistance and provide the vehicle with a suspension system, either completely or in part.

Two practical types of air-cushion suspension have been suggested—the plenum chamber and the peripheral jet. In the plenum chamber, air is pumped into a cavity formed

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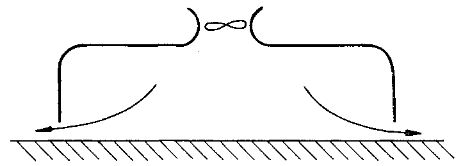

Fig. 1a Plenum chamber.

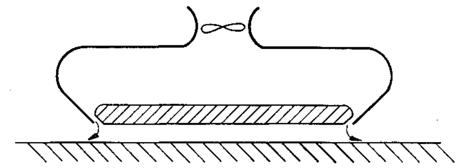

Fig. 1b Peripheral jet.

between the base of the vehicle and the track, and escapes around the periphery of the pad (Fig. 1a). The peripheral jet concept employs an air curtain to form a barrier across the gap between the track and the vehicle to impede the air escaping from the cushion (Fig. 1b). As it is proposed to support high-speed vehicles by these two systems, it is of interest to compare their sensitivity to the effect of forward speed. This can be done in a gross way by determining the vehicle speed at which air from the cushion is just prevented from escaping forward at the front of the support pad. The present note compares this "critical" velocity for two-dimensional peripheral jet and plenum chamber configurations. The justification for comparing the systems on a two-dimensional, rather than a three-dimensional basis is that the support pad configurations are usually either of high aspect ratio or effectively two-dimensional.

The prediction of the critical velocity for a two-dimensional peripheral jet was first determined in Ref. 4 by the relationship $\frac{1}{2}\rho V_f^2 = M_T/h$ where M_T had to be determined semi-empirically. This critical velocity can now be calculated using the simple forward speed theory of Alexander.[5]

Analysis

The magnitudes of the pressures and airflow velocities practically encountered are sufficiently small to allow the analysis to be reasonably performed treating the airflow as incompressible. Consider the front curtain of a two-dimensional peripheral jet vehicle travelling at the critical velocity, (Fig. 2). Following West[4] and Alexander,[5] it will be assumed that the static pressure on the upstream face of the air curtain is equal to the free-stream dynamic pressure. It will further be assumed that the static pressure at the nozzle plane is the average of the static pressure on either side of the air curtain. The validity of the latter assumption is examined in the discussion.

Therefore, applying Bernoulli's equation,

$$H = \tfrac{1}{2}\rho V^2 + \tfrac{1}{2}(p_c + \tfrac{1}{2}\rho V_f^2) \tag{1}$$

From the continuity equation

$$Vt = V_c t_c \tag{2}$$

and assuming that there are no flow energy losses between the nozzle exit and c

$$H = \tfrac{1}{2}\rho V_c^2 + p_c \tag{3}$$

From (1, 2, and 3)

$$\rho V_c^2 t_c = 2Ht\left\{\left[1 - \left(\frac{p_c}{H}\right)\right]\left[1 - \left(\frac{p_c + \frac{1}{2}\rho V_f^2}{2H}\right)\right]\right\}^{1/2} \tag{4}$$

The foregoing derivation follows that of the theory of Ref. 6, which is in reasonable agreement with experimental results.

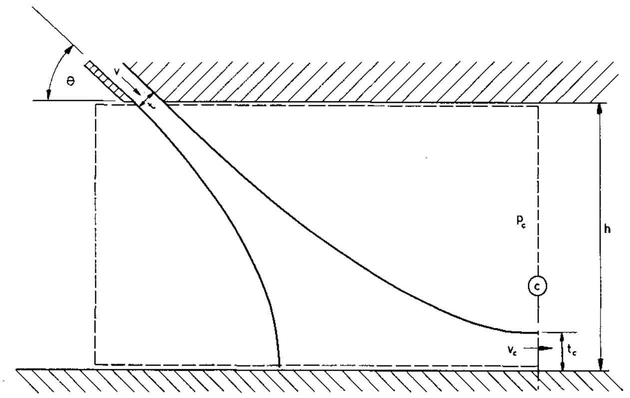

Fig. 2 Front jet at critical velocity.

Applying the principle of the conservation of momentum to the control volume shown in Fig. 2

$$(\tfrac{1}{2}\rho V_f^2 - p_c)h = \rho V_c^2 t_c - \rho V^2 t \cos\theta$$

and substituting for V from (1) and V_c from (4) yields

$$\tfrac{1}{2}\rho V_f^2 = p_c + 2H\frac{t}{h}\left(\left\{\left[1-\left(\frac{p_c}{H}\right)\right]\times\left[1-\left(\frac{p_c+\frac{1}{2}\rho V_f^2}{2H}\right)\right]\right\}^{1/2} - \cos\theta\left[1-\left(\frac{p_c+\frac{1}{2}\rho V_f^2}{2H}\right)\right]\right) \quad (5)$$

Now, the critical forward speed for the plenum chamber is achieved when the freestream dynamic pressure is equal to the cushion pressure. Therefore, from Eq. (5)

$$\psi = \left(\frac{\frac{1}{2}\rho V_f^2}{p_c}\right)^{1/2} = \left[1 + 2\left(\frac{H}{p_c}\right)\left(\frac{t}{h}\right)\times\left(\left\{\left[1-\left(\frac{p_c}{H}\right)\right]\left[1-\left(\frac{p_c+\frac{1}{2}\rho V_f^2}{2H}\right)\right]\right\}^{1/2} - \cos\theta\left[1-\left(\frac{p_c+\frac{1}{2}\rho V_f^2}{2H}\right)\right]\right)\right]^{1/2} \quad (6)$$

A peripheral nozzle angle of 45° is invariably chosen to minimize the power required to sustain the air curtain. Equation (6) is evaluated for $\theta = 45°$, using the functional relationship for (p_c/H) in terms of (h/t), θ, $[(\frac{1}{2}\rho V_f^2/H)]$ determined by Alexander.[5] These results are shown in Fig. 3 for the range of h/t of interest.

Discussion

From Fig. 3 it can be seen that the critical speed for a peripheral jet is slightly higher than that for a plenum chamber; i.e., the peripheral jet is slightly less sensitive to forward speed effects. From physical considerations, the critical speed for the peripheral jet can only occur when the freestream dynamic pressure is somewhat greater than the cushion pressure. Therefore, the values of ψ less than 1.0 [for $h/t = 3.0$] have no significance. They merely demonstrate the less satisfactory nature of simple jet performance theories for the smaller values of (h/t). This was to be expected, as can be seen from a comparison of the simple jet performance theory used in the present note with the "Exponential" theory of Elsley,[7] which is in better agreement with published experimental results.

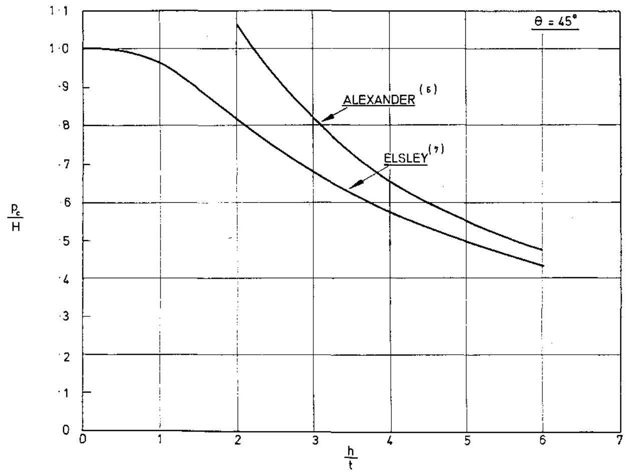

Fig. 4 Variation of cushion pressure with vehicle clearance.

It should be emphasized that the effect of forward speed is only one factor to be taken into account in making a rational choice between the peripheral jet and plenum chamber systems. The particular merits of the peripheral jet, all other things being equal, are that it requires less power, possesses greater stiffness in the heaving mode, and is more readily connected to a secondary air suspension element, as in the Hovercraft Development Ltd. "air spring." The principal virtue of the plenum chamber is its inherent simplicity.

The use of the term "critical" to denote the vehicle velocity at which air from the cushion is just prevented from escaping forward does not carry the connotation of "dangerous." The lift measured on a streamline two-dimensional peripheral jet pad[4] increased steadily with forward speed even for speeds 100 times that of the critical velocity. At speeds above the critical velocity, the air cushion acts in a comparable mode to a jet flap wing out of ground effect. Indeed, Foa[1] has suggested that operation above the critical velocity may be exploited for propulsion purposes, since the major portion of the air curtain momentum can be recovered as thrust.

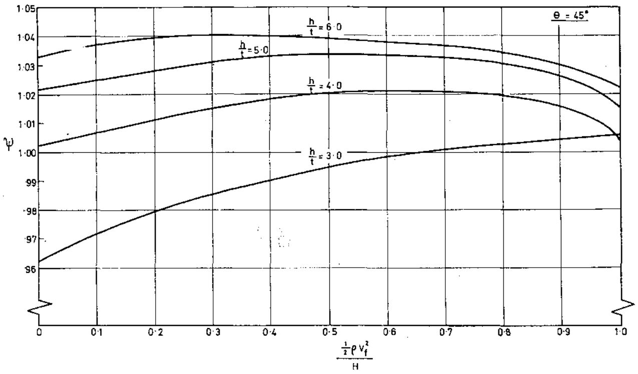

Fig. 3 Effect of forward speed on critical velocity ratio.

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